A note about this file

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Lord Rayleigh was one of the giants of Victorian physics. He developed much of the theories about the limitations of sharpness of optical systems, including pinholes. He was involved in a broad variety of problems and won the Nobel Prize in Physics in 1904 for his work on the behavior of gases and the discovery of the gas argon.

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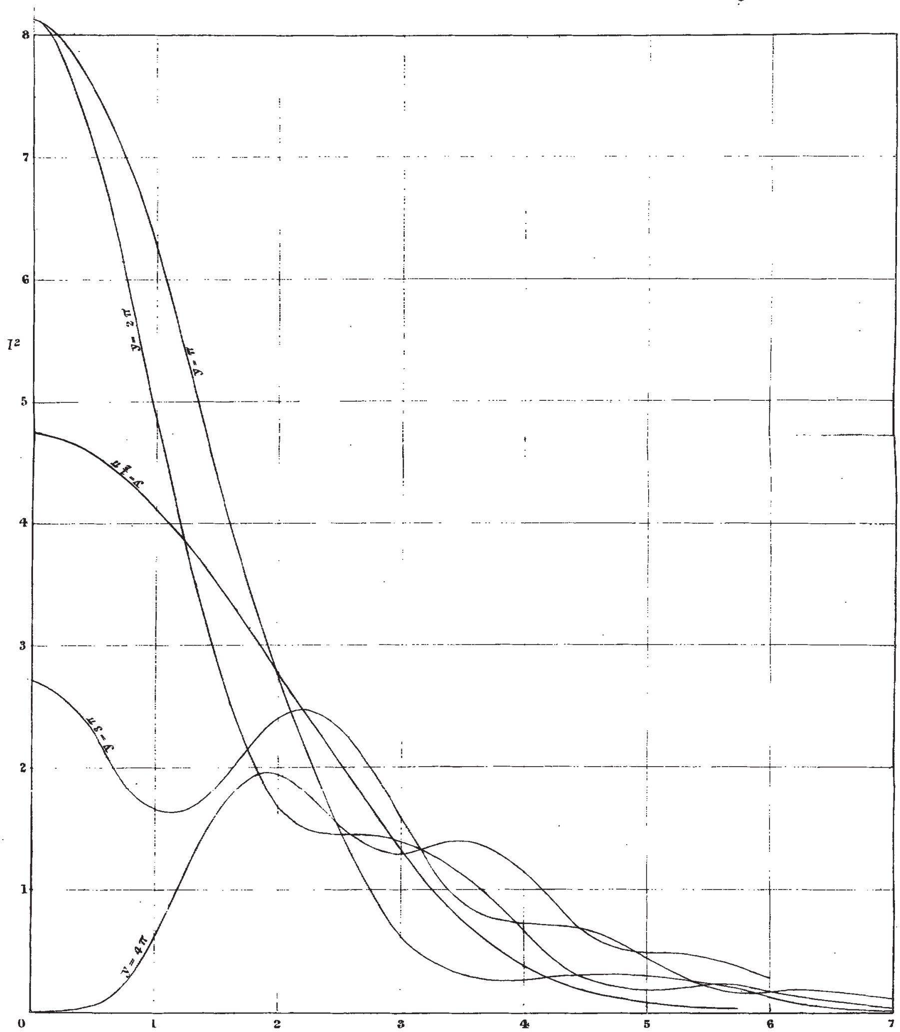

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X. *On Pin-hole Photography.*

By Lord RAYLEIGH, *Sec. R.S.**

[Plate IV.]

IT has long been known that the resolving power of lenses, however perfect, is limited, and more particularly that the capability of separating close distant objects, *e. g.* double stars, is proportional to aperture. The ground of the limitation lies in the finite magnitude of the wave-length of light (λ), and the consequent diffusion of illumination round the geometrical image of even an infinitely small radiant point. It is easy to understand the *rationale* of this process without entering upon any calculations. At the focal point itself all the vibrations proceeding from various parts of the aperture arrive in the same phase. The illumination is therefore here a maximum. But why is it less at neighbouring points in the focal plane which are all equally exposed to the vibrations from the aperture? The answer can only be that at such points the vibrations are discrepant. This discrepance can only enter by degrees; so that there must be a small region round the focus at any point of which the phases are practically in agreement, and the illumination sensibly equal to the maximum.

These considerations serve also to fix at least the order of magnitude of the patch of light. The discrepancy of phase is the result of the different distances of the various parts of the aperture from the eccentric point in question; and the greatest discrepancy is that between the waves which come from the nearest and furthest parts of the aperture. A simple calculation shows that the greatest difference of distance is expressed by $2rx/f$, where $2r$ is the diameter of the aperture, f the focal length, and x the linear eccentricity of the point under consideration. The question under discussion is at what stage does this difference of path introduce an important discrepancy of phase? It is easy to recognize that the illumination will not be greatly reduced until the extreme discrepancy of phase reaches half a wave-length. In this case

$$2x=f\lambda/2r,$$

which may be considered to give roughly the diameter of the patch of light. If there are two radiant points, the two representative patches will seriously overlap, unless the distance of their centres exceed $2x$. Supposing it to be equal to $2x$, which corresponds to an angular interval $2x/f$, we see that

* Communicated by the Author.

the double radiant cannot be resolved in the image, unless the angular interval exceed $\lambda/2r$.

Experiment* shows that the value thus roughly estimated is very near the truth for a rectangular aperture of width $2r$. If the aperture be of circular form, the resolving power is somewhat less, in the ratio of about 1·1 : 1.

It is therefore not going too far to say that there is nothing better established in optics than the limit to resolving power as proportional to aperture. On the other hand, the focal length is a matter of indifference, if the object-glass be perfect.

This is one side of the question before us. We now pass on to another, in which the focal length becomes of paramount importance.

"The function of a lens in forming an image is to compensate by its variable thickness the differences in phase which would otherwise exist between secondary waves arriving at the focal point from various parts of the aperture. If we suppose the diameter of the lens ($2r$) to be given, and its focal length (f) gradually to increase, these differences of phase at the image of an infinitely distant luminous point diminish without limit. When f attains a certain value, say f_1, the extreme error of phase to be compensated falls to $\frac{1}{4}\lambda$. Now, as I have shown on a previous occasion†, an extreme error of phase amounting to $\frac{1}{4}\lambda$, or less, produces no appreciable deterioration in the definition; so that from this point onwards the lens is useless, as only improving an image already sensibly as perfect as the aperture admits of. Throughout the operation of increasing the focal length, the resolving power of the instrument, which depends only upon the aperture, remains unchanged; and we thus arrive at the rather startling conclusion that a telescope of any degree of resolving power might be constructed without an object-glass, if only there were no limit to the admissible focal length. This last proviso, however, as we shall see, takes away almost all practical importance from the proposition.

"To get an idea of the magnitudes of the quantities involved, let us take the case of an aperture of $\frac{1}{5}$ inch, about that of the pupil of the eye. The distance f_1, which the actual focal length must exceed, is given by

$$\sqrt{\{f_1^2+r^2\}}-f_1=\tfrac{1}{4}\lambda;$$

so that [approximately]

$$f_1=2r^2/\lambda.$$

* "On the Resolving Power of Telescopes," Phil. Mag. August 1880.

† Phil. Mag. November 1879.

Thus, if $\lambda = 1/40{,}000$, $r = 1/10$,

$$f_1 = 800.$$

The image of the sun thrown on a screen at a distance exceeding 66 feet, through a hole $\frac{1}{5}$ inch in diameter, is therefore at least as well defined as that seen direct. In practice it would be better defined, as the direct image is far from perfect. If the image on the screen be regarded from a distance f_1, it will appear of its natural angular magnitude. Seen from a distance less than f_1, it will appear magnified. Inasmuch as the arrangement affords a view of the sun with full definition [corresponding to aperture] and with an increased apparent magnitude, the name of a telescope can hardly be denied to it.

"As the minimum focal length increases with the square of the aperture, a quite impracticable distance would be required to rival the resolving power of a modern telescope. Even for an aperture of four inches f_1 would be five miles"*.

A more practical application of these principles is to be found in landscape photography, where a high degree of definition is often unnecessary, and where a feeble illumination can be compensated by length of exposure. In a recent communication to the British Association † it was pointed out that a suitable aperture is given by the relation

$$2r^2 = f\lambda\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and a photograph was exhibited in illustration of the advantage to be derived from an increase of f. The subject was a weather-cock, seen against the sky, and it was taken with an aperture of $\frac{1}{16}$ inch, and at a distance of 9 feet. The amount of detail in the photograph is not markedly short of that observable by direct vision from the actual point of view. The question of brightness was also considered. As the focal length increases, the brightness (B) in the image of a properly proportioned pin-hole camera diminishes. For

$$B \propto r^2/f^2 \propto r^2\lambda^2/r^4 \propto \lambda^2/r^2 \propto \lambda/f. \quad . \quad . \quad . \quad (2)$$

There will now be no difficulty in understanding why a certain aperture is more favourable than either a larger or a smaller one, when f and λ are given. If the aperture be very small, the definition is poor even if the aid of a lens be

* "On Images formed without Reflection or Refraction," Phil. Mag. March 1881.

† Brit. Assoc. Report, 1889, p. 493.

invoked. If, on the other hand, the aperture be large, the lens becomes indispensable. The size of the aperture should accordingly be increased up to the point at which the lens is sensibly missed; and this, as we have seen, will occur in the neighbourhood of the value determined by (1). A more precise calculation can be made only upon the basis of a detailed knowledge of the distribution of light in the image.

The question of the best size of aperture for a pin-hole camera was first considered by Petzval*. His theory, though it can hardly be regarded as sound, brings out the failure of definition when the aperture is either too large or too small, and, as is very remarkable, gives (1) as the best relation between r, f, and λ. The argument is as follows:—If the hole be very small, the diameter of the patch of light representative of a luminous point is given by

$$D = f\lambda/r,$$

the measurement being made up to the first blackness in the diffraction-pattern. "This formula is only an approximate one, applicable when r is very small; in the case of a larger aperture, its diameter must be added to the value above given, that is to say,

$$D = 2r + f\lambda/r.$$

From the last formula we can at once deduce the best value for r; in other words, the size of the aperture which corresponds to the least possible value of D, and therefore to the sharpest possible image. In fact, differentiating the last expression, and setting in the ordinary manner, $dD/dr = 0$, we find at once

$$r = \sqrt{(\tfrac{1}{2}f\lambda)},$$

which corresponds to

$$D = 2\sqrt{(2f\lambda)}."$$

The assumption that intermediate cases can be represented by mere addition of the terms appropriate in the extreme cases of very large and very small apertures appears to be inadmissible.

The complete determination of the image of a radiant point as given by a small aperture is a problem in diffraction, solved only within the last years by Lommel†. In view of the

* *Wien. Sitz. Ber.* xxvi. p. 33 (1857); Phil. Mag. xvii. (1859) p. 1.

† "Die Beugungserscheinungen einer kreisrunden Œffnung und eines kreisrunden Schirmchens," *Aus den Abhandlungen der k. bayer. Akademie der Wiss.* ii. Cl. xv. Bd. ii. Abth. (München, 1884.)

practical application to pin-hole photography, I have thought that it would be interesting to adapt Lommel's results to the problem in hand, and to exhibit upon the same diagram curves showing the distribution of illumination in various cases. For the details of the investigation reference must be made to Lommel's memoir, or to the account of it in the *Encyclopædia Britannica*, art. "Wave Theory," p. 444. But it may be well to state the results somewhat fully.

In the following formulæ a is the distance from the radiant point to the aperture, b from the aperture to the screen upon which the image is formed. The circumstances being symmetrical about a line through the radiant point and the centre of the circular aperture (radius r), the illumination I^2 will be the same at all points of the screen equally distant ζ from the axis, and the problem to be solved is the determination of I^2 as a function of ζ for given values of a, b, r, and λ. Lommel finds that

$$I^2=\frac{1}{a^2b^2\lambda^2}(C^2+S^2), \quad . \quad . \quad . \quad . \quad (3)$$

where

$$C=\iint\cos\left(\tfrac{1}{2}\kappa\rho^2-l\rho\cos\phi\right).\rho\,d\rho\,d\phi, \quad . \quad . \quad . \quad (4)$$

$$S=\iint\sin\left(\tfrac{1}{2}\kappa\rho^2-l\rho\cos\phi\right).\rho\,d\rho\,d\phi, \quad . \quad . \quad . \quad (5)$$

and the following abbreviations are introduced:—

$$\frac{2\pi}{\lambda}\frac{a+b}{2ab}=\tfrac{1}{2}\kappa, \quad \frac{2\pi\zeta}{b}=l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The above corresponds to an incident wave whose intensity at the aperture is measured by $1/a^2$. The integration is to be taken over the area of the aperture, that is from $\phi=0$ to $\phi=2\pi$, and from $\rho=0$ to $\rho=r$. If we introduce the notation of Bessel's functions, we have

$$C=2\pi\int_0^r J_0(l\rho)\cos\left(\tfrac{1}{2}\kappa\rho^2\right).\rho\,d\rho, \quad . \quad . \quad . \quad . \quad (7)$$

$$S=2\pi\int_0^r J_0(l\rho)\sin\left(\tfrac{1}{2}\kappa\rho^2\right).\rho\,d\rho. \quad . \quad . \quad . \quad . \quad (8)$$

By integration by parts of these expressions Lommel develops series suitable for calculation. Setting

$$\kappa r^2=y \quad lr=z, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

he finds in the first place

$$C=\pi r^2\left\{\frac{\cos\frac{1}{2}y}{\frac{1}{2}y}U_1+\frac{\sin\frac{1}{2}y}{\frac{1}{2}y}U_2\right\}, \quad . \quad . \quad . \quad (10)$$

$$S=\pi r^2\left\{\frac{\sin\frac{1}{2}y}{\frac{1}{2}y}U_1-\frac{\cos\frac{1}{2}y}{\frac{1}{2}y}U_2\right\}, \quad . \quad . \quad . \quad (11)$$

where

$$U_1=\frac{y}{z}J_1(z)-\frac{y^3}{z^3}J_3(z)+\frac{y^5}{z^5}J_5(z)-\ldots, \quad . \quad . \quad (12)$$

$$U_2=\frac{y^2}{z^2}J_2(z)-\frac{y^4}{z^4}J_4(z)+\ldots. \quad . \quad . \quad . \quad . \quad (13)$$

These series are convenient when y is less than z.

The second set of expressions are

$$C=\pi r^2\left\{\frac{2}{y}\sin\frac{z^2}{2y}+\frac{\sin\frac{1}{2}y}{\frac{1}{2}y}V_0-\frac{\cos\frac{1}{2}y}{\frac{1}{2}y}V_1\right\}, \quad . \quad . \quad (14)$$

$$S=\pi r^2\left\{\frac{2}{y}\cos\frac{z^2}{2y}-\frac{\cos\frac{1}{2}y}{\frac{1}{2}y}V_0-\frac{\sin\frac{1}{2}y}{\frac{1}{2}y}V_1\right\}, \quad . \quad . \quad (15)$$

where

$$V_0=J_0(z)-\frac{z^2}{y^2}J_2(z)+\frac{z^4}{y^4}J_4(z)-\ldots, \quad . \quad . \quad (16)$$

$$V_1=\frac{z}{y}J_1(z)-\frac{z^3}{y^3}J_3(z)+\ldots. \quad . \quad . \quad . \quad . \quad (17)$$

These series are suitable when z/y is small.

When the primary wave is complete, $r=\infty$, and we have at once from the second set of expressions

$$C_\infty=\frac{2\pi}{\kappa}\sin\frac{l^2}{2\kappa}, \quad S_\infty=\frac{2\pi}{\kappa}\cos\frac{l^2}{2\kappa}, \quad . \quad . \quad . \quad (18)$$

so that

$$I^2_\infty=\frac{C^2_\infty+S^2_\infty}{a^2b^2\lambda^2}=\frac{1}{(a+b)^2}, \quad . \quad . \quad . \quad . \quad (19)$$

as we know it should be.

At the central point of the image where $z=0$, $V_0=1$, $V_1=0$,

$$C=\pi r^2\frac{\sin\frac{1}{2}y}{\frac{1}{2}y}, \quad S=\pi r^2\left\{\frac{2}{y}-\frac{\cos\frac{1}{2}y}{\frac{1}{2}y}\right\},$$

and

$$I^2=\frac{4}{(a+b)^2}\sin^2\left(\frac{\pi r^2}{\lambda}\frac{a+b}{2ab}\right). \quad . \quad . \quad . \quad . \quad (20)$$

In general by (10), (11),

$$C^2+S^2=\frac{4\pi^2r^4}{y^2}\{U_1^2+U_2^2\}=\pi^2r^4.M^2, \quad . \quad . \quad (21)$$

if with Lommel we set

$$M^2=\left(\frac{2}{y}U_1\right)^2+\left(\frac{2}{y}U_2\right)^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Also

$$I^2=\frac{\pi^2r^4}{a^2b^2\lambda^2}.M^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

In these formulæ U_1^2, U_2^2, and therefore by (22), (23) M^2 and I^2 are known functions of y and z. The connexion with r and ζ is given by the relations

$$y=\frac{2\pi r^2}{\lambda}\frac{a+b}{ab}, \quad z=\frac{2\pi r\zeta}{\lambda b}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

In Lommel's memoir are given the values of M^2 for integral values of z from 0 to 12 when y has the values π, 2π, 3π, &c. If we regard a, b, λ as given, each of these Tables affords a knowledge of the distribution of illumination as a function of ζ for a certain radius of aperture by means of the two equations (24). In each case ζ is proportional to z; but in comparing one case with another we have to bear in mind that the ratio of ζ to z varies. As our object is to compare the distributions of illumination when the aperture varies, we must treat ζ, and not z, as the abscissa in our diagrams. Another question arises as to how the scale of the ordinate I^2 should be dealt with in the various cases. If we take (23) as it stands we shall have curves corresponding to the same actual intensity of the radiant point. For some purposes this might be desirable; but in the application to photography the deficiency of illumination when the aperture is much reduced would always be compensated by increased exposure. It will be more practical to vary the scale of ordinates from that prescribed in (23), so as to render the illumination corresponding to an extended source of light, such as the sky, the same in all cases. We shall effect this by removing from the right-hand member of (23) a factor proportional to the area of aperture, proportional that is to r^2, or y. Thus for any value of y equal to $s\pi$, we shall require to plot as ordinate, not M^2 simply, but sM^2, and as abscissa, not z simply, but $z/\sqrt{s}$. The following are at once deduced from Lommel's tables III.–VI.

$y=\pi$.

z.	$z/\sqrt{1}=x$.	M^2.
0	0	·8106 Max.
1	1	·6286
2	2	·2772
3	3	·0623
4	4	·0269
5	5	·0306
6	6	·0121
7	7	·0018
8	8	·0051
9	9	·0037
10	10	·0004
11	11	·0013
12	12	·0016
3·8317	......	·0263 Min.
4·7153	......	·0320 Max.
7·0156	......	·0018 Min.
8·3060	......	·0055 Max.
10·1735	......	·0003 Min.
11·5785	......	·0019 Max.

$y=2\pi$.

z.	$z/\sqrt{2}$.	$2M^2$.
0	·000	·8106 Max.
1	·707	·6316
2	1·414	·3117
3	2·121	·1560
4	2·829	·1438
5	3·536	·1077
6	4·243	·0426
7	4·950	·0200
8	5·657	·0227
9	6·364	·0125
10	7·071	·0034
11	7·778	·0053
12	8·485	·0046
3·5977	2·544	·1440 Min.
3·8317	2·710	·1442 Max.
7·0156	4·961	·0198 Min.
7·8879	5·578	·0229 Max.
10·1735	7·193	·0032 Min.
11·4135	8·070	·0059 Max.

$y=3\pi$.

z.	$z/\sqrt{3}$.	$3M^2$.
0	·000	·2702 Max.
1	·577	·2159
2	1·154	·1631
3	1·732	·2110
4	2·310	·2440
5	2·887	·1734
6	3·464	·0916
7	4·041	·0730
8	4·619	·0651
9	5·195	·0335
10	5·773	·0156
11	6·350	·0178
12	6·927	·0122
1·9960	1·153	·1631 Min.
3·8317	2·212	·2467 Max.
7·0156	4·050	·0737 Min.
7·0878	4·092	·0739 Max.
10·1735	5·871	·0154 Min.
11·0361	6·374	·0178 Max.

$y=4\pi$.

z.	$z/2$.	$4M^2$.
0	·000	·0000 Min.
1	·500	·0056
2	1·000	·0609
3	1·500	·1594
4	2·000	·1947
5	2·500	·1515
6	3·000	·1293
7	3·500	·1399
8	4·000	·1148
9	4·500	·0658
10	5·000	·0484
11	5·500	·0458
12	6·000	·0280
3·8317	1·9158	·1961 Max.
5·8078	2·9489	·1291 Min.
7·0156	3·5078	·1399 Max.
10·1735	5·0867	·0483 Min.
10·3861	5·1930	·0483 Max.

As it appeared desirable to trace the curve corresponding to a smaller value of y than any given by Lommel, I have calculated by means of (12), (13) the value of $\frac{1}{2}M^2$, that is of

$$\frac{8}{\pi^2}(U^2_1+U_2^2),$$

corresponding to $z=0, 1, 2, 3, 4$.

The results are as follows:—

$y=\frac{1}{2}\pi$.

z.	$\sqrt{2}.z$.	$\frac{1}{2}M^2$.
0	·000	·4748
1	1·414	·3679
2	2·828	·1590
3	4·243	·0272
4	5·657	·0041

The various curves, or rather the halves of them, are plotted in Plate IV., and exhibit to the eye the distribution of light in the images corresponding to the different apertures. It is at once evident that $y=\frac{1}{2}\pi$ is too small, and that $y=3\pi$ is too great. The only question that can arise is between $y=\pi$ and $y=2\pi$. The latter has decidedly the higher resolving power, but the advantage is to some extent paid for in the greater diffusion of light outside the image proper. In estimating this we must remember that the amount of light is represented, not by the *areas* of the various parts of the diagrams, but by the *volumes* of the solids formed by the revolution of the curves round the axis of I^2. In virtue of the method of construction the total volume is the same in all cases. The best aperture will thus depend in some degree upon the subject to be represented; but there is every reason to think that in general $y=2\pi$ will prove more advantageous than $y=\pi$. It will be convenient to recall that

$$y/\pi=\frac{2r^2}{\lambda}\frac{a+b}{ab};$$

or, if we write $a=\infty$, $b=f$,

$$y/\pi=2r^2/\lambda f. \qquad (25)$$

The curve $y=\pi$ thus corresponds to (1); and we conclude that the aperture may properly be somewhat enlarged so as to make

$$r^2=\lambda f. \qquad (26)$$

In the general case when a is finite, y/π represents four times the number of wave-lengths by which the extreme ray is retarded relatively to the central ray; for

$$\frac{\sqrt{(a^2+r^2)}+\sqrt{(b^2+r^2)}-a-b}{\lambda}=\frac{r^2}{2\lambda}\frac{a+b}{ab}, \text{ approximately.}$$

According to (26) the aperture should be enlarged until the retardation amounts to $\frac{1}{2}\lambda$.

In the image of a double star the curves of brightness proper to the two components are superposed. If the components are equal, resolution will be just beginning when the distance of the geometrical images asunder is the double of the value of ζ for which I^2 has about one-half its maximum value. By inspection of the curve for $y=2\pi$ we see that there will not be much appearance of resolution until $z/\sqrt{2}=1\cdot5$. The corresponding angular interval between the two components is

$$\frac{2\zeta}{f}=\frac{1\cdot5\times\sqrt{}}{\pi}\sqrt{\left(\frac{\lambda}{f}\right)}. \quad . \quad . \quad . \quad . \quad (27)$$

This may be regarded as defining the maximum separating power as a function of λ and f.

Passing on from the theoretical discussion, I will now describe certain laboratory observations upon the defining power of various apertures. A succession of such, of gradually increasing magnitude, were perforated in a piece of thin sheet zinc, and were measured under the microscope. The diameters, in fractions of an inch, are as follows:—

(1)	(2)	(3)	(4)	(5)	(6)
·0210,	·0240,	·0262,	·0290,	·0326,	·0366.

The objects, whose images were examined, are (1) a grating cut out of sheet zinc, and (2) a pair of equal round holes a quarter of an inch apart. The period of the grating is also a quarter inch, and the transparent and opaque parts are equally wide. Behind the grating, or double hole, was placed a paraffin lamp and a large condensing lens. The distance a between the objects and the apertures under test was about 18 feet.

The best image with a given aperture is obtained by bringing the eye immediately behind, without the use of a focusing lens. But the image formed at a sufficient distance beyond, and examined with a focusing glass of low power, is nearly as good. Thus at a sufficient distance (b) the largest aperture gives the best image, but at a *given* distance behind the case is otherwise. For example, when the image was formed at 8 inches distance, (2) and (3) were about equal as respects the double hole, while (1) was decidedly inferior, and that not apparently from want of light. In the case of the grating (3) had perhaps the advantage over (2).

A photograph of the double hole was now taken under the same circumstances with an exposure of 80 minutes. Aperture (2) was here decidedly better than (3), and (1) was almost as good as (2). The (negative) image formed by (5) exhibited a pair of white spots near the centre of a patch of black, corresponding to the state of things indicated in the curve $y=4\pi$. The difference between the photograph and the result obtained by eye observation is readily explained by the smaller effective wave-length in the former case.

The difference just spoken of is intensified when the light is white. In one experiment upon cloud-light $a=21$ feet, $b=10$ inches. In the resulting photograph, obtained upon an Ilford plate with an exposure of 30 minutes, the image from (2) was decidedly the best.

We may utilize the last result to calculate the relation between aperture and focus most suitable for out of door photography. We have

$$(2r)^2\left(\frac{1}{a}+\frac{1}{b}\right)=(\cdot 0240)^2\left(\frac{1}{252}+\frac{1}{10}\right)$$

$$=10^{-5}\times 5\cdot 99 \text{ inches}=10^{-4}\times 1\cdot 52 \text{ centim.}$$

Thus, if $a=\infty$, as may usually be supposed in landscape-photography, the most favourable diameter of aperture is given by

$$(2r)^2/f=10^{-5}\times 6\cdot 0 \text{ inches}=10^{-4}\times 1\cdot 5 \text{ centim.},$$

the first number being employed if r and f are measured in inches, and the latter when the measures are in centimetres*. If $f=12$ inches, $2r=\cdot 027$ inch. If $f=7\times 12=84$ inches, $2r=\cdot 071$ inch.

The experimental determination of the best value of y is more easily effected by eye observations. In order to render the wave-length more definite an orange-red glass was employed. With $a=18$ feet, $b=8$ inches, the image formed by aperture (3) was judged to be decidedly the best, (2) was

* The effect of varying the diameter of aperture in photographic landscape work has been tested by Capt. Abney; but I am not in possession of the data as to focal length necessary for a comparison with the above.

next, while (1) and (4) were decidedly behind. Thus we may take as the most favourable aperture $2r = \cdot 026$ inch.

The mean wave-length of the light employed was found with the aid of a grating by a comparison with a soda flame :—

$$\text{Mean } \lambda : \lambda_D = 239 : 226 ;$$

so that

$$\lambda = \frac{239}{226} \times 5\cdot 89 \times 10^{-5} = 6\cdot 23 \times 10^{-5} \text{ centim.}$$

Hence

$$y/\pi = \frac{2r^2}{\lambda}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{(\cdot 026)^2}{2}\frac{7}{54}\frac{2\cdot 54 \times 10^5}{6\cdot 23} = 1\cdot 79,$$

agreeing very well with what was expected from the curves.

If we now assume that the best value of y is 1·8, we can calculate backwards from the photographic results to find the mean λ there effective. We have

$$10^{-4} \times 1\cdot 52 = (2r)^2\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{2\lambda y}{\pi} = 2\lambda \times 1\cdot 8;$$

whence

$$\lambda = 4\cdot 2 \times 10^{-5} \text{ centim.},$$

a little less than that belonging to Fraunhofer's line G.

To test the improvement of definition which according to (27) accompanies an increase of f, I have used an aperture of ·07 inch and a focal length of 7 feet. The aperture was perforated in sheet zinc, and was placed in the shutter of a room otherwise completely darkened. The subject was a group of cedars, and, being somewhat dark in the shadows, required an exposure of about an hour and a half, even in sunshine. The resulting 12 × 10-inch photographs fully bear out expectations. To appear in natural magnitude the pictures would of course need to be held at 7 feet distance from the eye ; but even at 3 or 4 feet the apparent definition is sufficient. I have also taken panoramic pictures with an aperture of ·027 inch and a focal distance of 12 inches ; but in this case there is nothing that could not equally well be done with an ordinary portable camera.

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Dec. 2, 1890.